

Esercizi Teorema di Green

1) Ese

$$\int_C y^2 dx + x dy$$

è parametrizzato $C = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$

$$\omega = (y^2, x, 0)$$

$$\iint_D (1 - 2y) dx dy \Rightarrow \begin{cases} x = \cos \theta & \theta \in [0, 2\pi] \\ y = \sin \theta & \theta \in [0, 2\pi] \end{cases}$$

$$\iint_D (1 - 2y) dx dy = \int_0^{2\pi} \int_0^1 (1 - 2 \sin \theta) r dr d\theta = \int_0^{2\pi} \left(\frac{1}{2} r^2 - 2r^2 \sin \theta \right) \Big|_0^1 d\theta =$$

$$= \int_0^{2\pi} \left(\frac{1}{2} - 2 \sin \theta \right) d\theta = \pi$$

Ex 6

$$\int_{\gamma} F \, dP$$

$$F(x, y) = (x^2 y^3, y)$$

$$\gamma \text{ param. } A = \{(x, y) \in \mathbb{R}^2; x^2 + y^2 \leq 4\}$$

$$\int_{\gamma} x^2 y^3 \, dx + y \, dy$$

$$\iint_D -3x^2 y^2 \, dx \, dy$$

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$$

$$-3 \int_0^{2\pi} \int_0^2 r^5 \cos^2 \theta \sin^2 \theta \, dr \, d\theta$$

$$\dots (\cos \theta \sin \theta)^2 = \frac{\sin^2 2\theta}{4}$$

$$-\frac{3}{4} \int_0^{2\pi} \sin^2(2\theta) d\theta$$

$$-\frac{3}{4} \left[\frac{\theta}{2} - \frac{\sin 4\theta}{8} \right]_0^{2\pi}$$

$$-\frac{3}{4} \left(\frac{64}{2} - \frac{1}{8} \right) (\pi)$$

$$-\frac{63\pi}{8}$$

$$(\sin 2\theta)^2 = \frac{1 - \cos(4\theta)}{2}$$

$$\left(\frac{\theta}{2} - \frac{\sin 4\theta}{8} \right)$$